## Discrete Mathematical Structure Final Exam (Spring 2013)

No:
Name:

## GRAPHS

1. (15P) Use Dijkstra's algorithm to find the length of a shortest path and a shortest path from $\mathbf{a}$ to $\mathbf{z}$ in the following weighted graph (show each iteration in below box).


| a | b | c | d | e | f | g | h | i | j | z |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | - | - | - | - | - | - | - | - | - | - |
| 0 | 4 | - | - | 7 | 5 | - | - | - | 2 | - |
| 0 | 4 | 7 | 10 | 7 | 5 | 11 | - | 8 | 2 | - |
| 0 | 4 | 7 | 9 | 7 | 5 | 11 | 13 | 8 | 2 | - |
| 0 | 4 | 7 | 9 | 7 | 5 | 11 | 13 | 8 | 2 | 12 |

The length of the shortest path is 12 .
2. $(10 P)$ For the following graph, determine if it has an Euler path that is not a circuit.


TREES
It has an Euler trail that is not a circuit because it has two vertices with odd degree ( $b$ and e).
3. (15P) Construct an optimal Huffman code for the set of letters in the table. Find the average length of bit strings encoding 39 -letter words with the Huffman code.

| letter | frequency |
| :---: | :---: |
| A | 2 |
| B | 12 |
| C | 3 |
| D | 18 |
| E | 4 |

 A:0001
4. In the following graph with its vertices in alphabetical order find a spanning tree using
i. (10P) Breadth-first search.
ii. (10P) Depth-first search.


## AUTOMATA

1. According to finite state automaton transition diagram given on the right,
i. (10P) Design the grammar rules.
ii. (10P) Describe acceptable strings as a sentence.

$$
\begin{aligned}
& \text { i. } A \rightarrow a B \mid b C B \rightarrow b F C \rightarrow a F \\
& F \rightarrow a B|b C| \lambda
\end{aligned}
$$

Then by removing $\lambda$,

$$
\begin{array}{ll}
A \rightarrow a B \mid b C & B \rightarrow b F \mid b \\
C \rightarrow a F \mid a & F \rightarrow a B \mid b C
\end{array}
$$

## ii.

The strings which start and finish with the different symbols, where the same letters cannot be consecutive, are accepted.
2. Let G be the grammar of a language with non terminal symbols $\{\mathrm{E}, \mathrm{F}\}$, terminal symbol $\{\mathrm{a}, \mathrm{b},+$, *\}, starting symbol $E$, and rules as follow.

$$
\begin{array}{ll}
E \rightarrow F & E \rightarrow+F E \\
E \rightarrow F F E & F \rightarrow a \\
F \rightarrow b &
\end{array}
$$

i. $(10 P)$ Find a derivation for the string " $*+a * b b a$ ".
ii. (10P) Draw its deterministic finite state automaton transition diagram by using nondeterministic one.
i.

It cannot be possible to derivate that string " $*+a * b b a$ " by using G grammar.

(then deterministic one)


