Discrete Mathematics Midterm Exam (Spring 2016)

No :
Name:

1. LOGIC (20P) Show that the hypotheses below lead to the conclusion: We will be home by the sunset.
   - It is not sunny this afternoon and it is colder than yesterday.
   - We will go swimming only if it is sunny.
   - If we do not go swimming, then we will take a canoe trip. □ If we take a canoe trip, then we will be home by sunset.

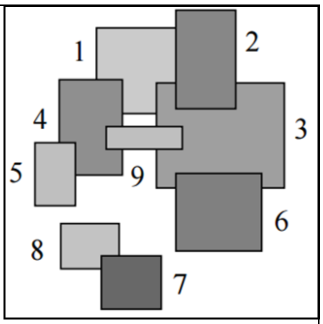
p : "It is sunny this afternoon."
r : "We will go swimming."
q : "It is colder than yesterday."
s : "We will take a canoe trip."
t : "We will be home by sunset."

Hypothesis: $\neg p \wedge q$, $r \rightarrow p$, $\neg r \rightarrow s$, $s \rightarrow t$

Conclusion: t

1.	$\neg p \wedge q$	1st hypothesis
2.	$\neg p$	Simplification using step 1
3.	$r \rightarrow p$	2nd hypothesis
4.	$\neg r$	Modus tollens using steps 2 & 3
5.	$\neg r \rightarrow s$	3rd hypothesis
6.	s	Modus ponens using steps 4 & 5
7.	$s \rightarrow t$	4th hypothesis
8.	t	Modus ponens using steps 6 & 7

2. RELATION (20P) Let intersections between rectangles be the relation. According to figure at the right, at first write relations as a partial order set, and then draw Hasse diagram of it. (Note: Of course all rectangles intersect to itself. And (1, 2) means that "Rectangle 1" and "Rectangle 2" overlap.)



S={(1,1),(1,2),(1,3),(1,4),(2,2),(2,3),(3,3),(3,6),(3,9),(4,4),(4,5),(4,9),
(5,5),(6,6),(7,7),(7,8),(8,8),(9,9)}

1. Reflexive is available.
2. Anti-symmetric is available (by means of writing concept).
3. Transitive is not available, i.e., (1, 3) AND (3, 9) are present, but (1, 9) is absent

Therefore, the Hasse diagram cannot be drawn here.

3. CHINESE REMAINDER (20P) Find x by solving simultaneously the following three congruencies.

$x \equiv 2 \ (mod\ 7)$
$x \equiv 3 \ (mod\ 5)$
$x \equiv 1 \ (mod\ 13)$

$m_1 = 7$	$m_2 = 5$	$m_3 = 13$	$M = 7 \text{ x } 5 \text{ x } 13 = 455$
$r_1 = 2$	$r_2 = 3$	$r_3 = 1$	
$M_1 = 65$	$M_2 = 91$	$M_3 = 35$	
$s_1 = \text{inv}(M_1 \bmod 7) = \text{inv}(2) = 4$	$s_2 = \text{inv}(1) = 1$	$s_3 = \text{inv}(9) = 3$	

$x = M_1 s_1 r_1 + M_2 s_2 r_2 + M_3 s_3 r_3 = 65\text{x}4\text{x}2 + 91\text{x}1\text{x}3 + 35\text{x}3\text{x}1 = 520+273+105 = 898 \ (\text{mod } 455)$

$x = 443 + 455\ k$

4. INDUCTION (20P) Using mathematical induction, prove that $n^3 - n$ is a multiple of 3 for $\forall n \geq 0$.

$n3 - n = 3\ k_1$

1. Basis step. For both $n = 0$ and $n = 1$, is available
2. Induction step. For $n \rightarrow n + 1$,
   $(n+1)^3 - n = 3k_2$
   $n^3 + 3n^2 + 2n = 3k$ here, we can reduce the equation by using ($n3 - n = 3\ k_1$)
   $3n\ (n + 1) = 3k_3$

   Because this difference is reasonable, we can say that the hypothesis is proved by induction.

5. PIGEONHOLE (20P) Among six people where each pair are either friends or enemies, show that there are 3 people either friends or enemies (or both) to each other.

Let these six persons be {A, B, C, D, E, F}. Now, we can think about person A. By the pigeonhole principle, of the remaining 5 people, either 3 or more are friends of A or 3 enemies of A. Suppose first that B, C, and D are friends of A. If any two of these persons are friends, these 2 and A form a group 3 mutual friends. If none of B, C, and D are friends, then these three persons form a group of 3 mutual enemies.