Discrete Mathematics Midterm Exam (Spring 2017)

No :

Name:

1. LOGIC (20*P*) Consider these statements:

   P(x): “x is a hummingbird”

   R(x): “x is large”

   S(x): “x lives on honey”

   Q(x): “x is richly colored”

Express the statements below using quantifiers and P(x), Q(x), R(x), S(x).

a. (5P) “All hummingbirds are richly colored.”

   $\forall x(P(x) \rightarrow Q(x))$.

b. (5P) “No large birds live on honey.”

   $\neg\exists x(R(x) \land S(x))$.

c. (5P) “Birds that do not live on honey are dull in color.”

   $\forall x(\neg S(x) \rightarrow \neg Q(x))$.

d. (5P) “Hummingbirds are small.”

   $\forall x(P(x) \rightarrow \neg R(x))$.

2. RELATION (20*P*) By checking whether the set S given below is a partial order set, draw its Hasse diagram.

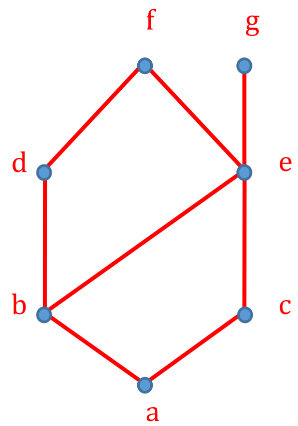
```
S={(a,a),(a,b),(a,c),(a,d),(a,e),(a,f),(a,g),(b,b),(b,d),(b,e),(b,f),(b,g),
   (c,c),(c,e),(c,f),(c,g),(d,d),(d,f),(e,e),(e,f),(e,g),(f,f),(g,g)}
```

At first, we should check the three rules:

1. Reflexive: All elements are related itself, so it is OK.
   (a,a),(b,b),(c,c),(d,d)(e,e),(f,f),(g,g)

2. Anti-symmetric: In the set S, found relations in case both (x,y) and (y,x) are only the same ones in previous rule. For this reason, it is OK.

3. Transitive: We can find all transitive situations
   (a,b)-(b,e)→(a,e)
   (b,d)-(d,f)→(b,f)
   (c,e)-(e,g)→(c,g)
   (a,d)-(d,f)→(a,f)
   ...



3. COMPLEXITY (20*P*) Find the complexity of the algorithm below for the worst case.

```
function xR(n)
      z = 0;
      for i=1 to n
            z = z + i;
      end for
      if n==1
            return 1;
      else
            return z+xR(round(n/2));
      end if
end function
```

The worst case is represented by big O.

$$O(T) = O\left(n + \frac{n}{2} + \frac{n}{4} + \cdots + \frac{n}{n}\right)$$
$$= O\left(n\left(1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{n}\right)\right)$$
$$= O\left(n\left(\frac{1-\left(\frac{1}{2}\right)^{1+log_2 n}}{1-\frac{1}{2}}\right)\right)$$
$$= O\left(n\left(2 - 2^{-log_2 n}\right)\right) = O(2n-1)$$
$$= \mathbf{O}(\boldsymbol{n})$$

4. RECURSIVE RELATION (20*P*) Write the mathematical expression of the algorithm "xF" down independent of the term of previous value.

```
function xF(n)
      if n==1
            return 1;
      else
            return 3 * xF(n-1) + 1;
      end if
end function
```

$$a_n = 3a_{n-1} + 1 = 3(3a_{n-2} + 1) + 1 = 3^{n-1}a_1 + (1 + 3 + 3^2 + \cdots + 3^{n-2})$$
$$= 3^{n-1}a_1 + \frac{1-3^{n-1}}{1-3} = \frac{(2a_1 + 1) * 3^{n-1} - 1}{2}$$

$$xF(n) = \begin{cases} 1 & , n = 1 \\ \dfrac{3^n - 1}{2} & , other \end{cases}$$

5. BAYES (20*P*) At a city, 4% of men and 1% of women are over 190cm tall. The total student population is divided in the ratio m:2/w:3. If a student is selected at random from among all those over 190cm tall, by computing probabilities, comment the probable gender of that student.

$$P(T \mid W) = 0.01$$
$$P(T \mid M) = 0.04$$
$$P(M) = 0.4$$
$$P(W) = 0.6$$
$$P(T) = 0.4 * 0.04 + 0.6 * 0.01 = 0.022$$

$$P(W \mid T) = \frac{P(T|W)P(W)}{P(T)} = \frac{0.01 * 0.6}{0.022} = \frac{3}{11}$$

$$P(M \mid T) = \frac{P(T|M)P(M)}{P(T)} = \frac{0.04 * 0.4}{0.022} = \frac{8}{11}$$

Because of $P(M|T) > P(W|T)$, one selected at random in tall students is probable a man.

NOTE: In any question, you can use this: $(1+r+r^2+r^3+\ldots+r^n)=(1-r^{n+1})/(1-r)$