1. **(50 P)** Transform NFA at the right into DFA form. ($\Sigma = \{a, b\}$)

We should start at preparing power set of $Q_1$ (set of states of the DFA).

$Q_2 = \{\emptyset, q_1, q_2, q_3, q_{12}, q_{13}, q_{23}, q_{123}\}$

Then we can define new final states for $Q_2$

$F = \{q_2, q_3, q_{12}, q_{13}, q_{23}, q_{123}\}$ because they includes $q_2$ or $q_3$

and start state

$q = q_{123}$ because it can pass $q_1$ and $q_3$ by $\epsilon$ transition

Lastly, transition function

\[
\begin{align*}
\delta(\emptyset, a) &= \emptyset \\
\delta(q_1, a) &= q_3 \\
\delta(q_2, a) &= q_3 \\
\delta(q_3, a) &= q_{123} \\
\delta(q_{12}, a) &= q_3 \\
\delta(q_{13}, a) &= q_{123} \\
\delta(q_{23}, a) &= q_{123} \\
\delta(q_{123}, a) &= q_{123}
\end{align*}
\]

2. **(50 P)** When all $\epsilon$ transitions are deleted from the NFA at the top, the system transforms a DFA. For this new DFA, write the regular expression down. ($\Sigma = \{a, b\}$)

First, each state should be considered as an independent language

$L_{q_1} = a.L_{q_2} \cup b.L_{q_2}$

$L_{q_2} = \epsilon \cup a. L_{q_3} \cup b. L_{q_3}$

$L_{q_3} = \epsilon \cup a. L_{q_2} \cup b. L_{q_1}$

We should organize $L_{q_1}$

$L_{q_1} = a. (\epsilon \cup a. L_{q_2} \cup b. L_{q_1}) \cup b. L_{q_2} = ab. L_{q_1} \cup (b \cup aa). L_{q_2} \cup a$

Because $L_{q_1}$ depend on $L_{q_2}$, we should reduce $L_{q_2}$

$L_{q_2} = \epsilon \cup (a \cup b). (\epsilon \cup a. L_{q_2} \cup b. L_{q_1}) = (aa \cup ba). L_{q_2} \cup (ab \cup bb). L_{q_1} \cup a \cup b$

$L_{q_2} = (aa \cup ba)^* ((ab \cup bb). L_{q_1} \cup a \cup b)$

Now we can reorganize $L_{q_1}$ by placing $L_{q_2}$

$L_{q_1} = (ab \cup (b \cup aa)(aa \cup ba)^* (ab \cup bb)). L_{q_1} \cup (b \cup aa)(aa \cup ba)^* (a \cup b) \cup a$

$L_{q_1} = (ab \cup (b \cup aa)(aa \cup ba)^* (ab \cup bb))^*((b \cup aa)(aa \cup ba)^*(a \cup b) \cup a)$